Show Inverse z-transform using polynomial division
X(z) =N(z)/D(z) where N(z) and D(z) are polynomials.
then polynomial division can be used to put X(z) into a polynomial form and the inverse z-transform found be matching terms.

Example 1:

$$X(z)=\frac{1}{1-az^{-1}}$$

$$1-az^{-1}\overline{)\;1}\quad 1+az^{-1}+a^2z^{-2}....$$

$$1-az^{-1}$$

$$az^{-1}-a^2z^{-2}$$

$$a^2z^{-2}$$

By long division

$$X(z)=1+az^{-1}+a^2z^{-2}+a^3z^{-3}...$$

Matching terms

x[n]={1, a, $a^2, a^3...$}

Example 2:

$$X(z)=\frac{z^{-1}+z^{-2}}{1-3z^{-1}+3z^{-2}+z^{-3}}$$

$$1-3z^{-1}+3z^{-2}+z^{-3}\overline{)\;z^{-1}+z^{-2}}\quad z^{-1}+4z^{-2}+9z^{3}$$

$$z^{-1}-3z^{-2}+3z^{-3}+z^{-4}$$

$$4z^{-2}+3z^{-3}-z^{-4}$$

$$4z^{-2}-124z^{-3}+14z^{-4}-z^{-5}$$

By long division

$$X(z)=z^{-1}+4^2z^{-2}+9^3z^{-3}...$$

Matching terms

x[n]={0, 1, 4, 9.....} $= n^2$